Supplementary Materials

Cohesive subgroups analysis is a powerful and mathematically rigorous method to characterize network robustness. The strength lies in the capacity to detect strong connections among nodes that not only have no neighbors in common, but that may be distantly separated in the graph (*1*).

Cliques

A clique is a subgroup of actors in which each actor is adjacent to any other actors in it, and it is impossible to add any other actors to the clique without violation of this condition (*2*). In our study, we constrain the minimum size of any clique to three.

When there are many cliques, it is difficult to interpret the result of cohesive subgroups for the overlap between cliques, which can result in hidden features of the structure. A method to solve this issue would be to try to remove or reduce the overlap by performing additional analysis such as clique co-membership(*2*). The first step is to combine cliques who have more than 2/3 of all actors being shared. After the first step, if there are still too many cliques, those that share more than 1/3 of the same members can be merged (*3*). From a small number of cliques, we can detect a set of key nodes acting as the bridge between subgroups.

Lambda Sets

Lambda sets, based on the property that members of the set have greater edge connectivity with other members than with non-members, have been shown to correspond to a particular hierarchical clustering of the nodes in a network (*4*). It is a maximal subset of actors who have more edge-independent paths connecting them to each other than to outsiders since actors in lambda sets with connectivity λ have a minimum of λ independent paths linking any one to any other. When λ is large, a lambda set describes a subset that is relatively difficult to disconnect by means of edge removals (*4*). In infectious disease research, we can detect those who are the most active in the subgroup, which is the most important for disease control.

SUPPLEMENTARY TABLE S1. Clique analysis in HIV transmission network with 184 nodes in Guangzhou, Guangdong Province, China, 2015–2017.

Cliques	Number of nodes	ID
1	24	10 14 15 20 26 27 34 37 4 41 62 80 R10 R3 M003 M004 M010 M011 M050 M057 M060 M064 M100 M107
2	22	14 15 20 26 27 34 37 4 59 62 80 R3 M003 M004 M010 M011 M050 M057 M060 M064M100M107
3	22	14 15 20 26 27 34 37 4 47 59 80 9 M003 M004 M010 M011 M050 M057 M060 M064M100M107
4	24	10 14 15 20 26 27 34 37 4 41 62 80 R10 R3 M003 M004 M010 M011 M013 M050 M057 M060 M064 M100
5	22	14 15 20 26 27 34 37 4 59 62 80 R3 M003 M004 M010 M011 M013 M050 M057 M060 M064 M100
6	17	10 20 26 27 34 4 41 62 R10 R3 M003 M010 M017M050 M057 M060 M107
7	12	2 20 26 27 4 59 62 R3 M013 M050 M057 M060
8	5	34 4 41 M057 M073
9	4	25 M101 M103 M104
10	6	30 75 R12 M026 M048 M056
11	4	30 M026 M037 M065
12	5	75 R12 M026 M056 M108
13	5	8 R12 M026 M056 M109
14	3	M026 M048 M068

SUPPLEMENTARY TABLE S2. Lambda sets in HIV transmission network with 184 nodes in Guangzhou, Guangdong Province, China, 2015–2017.

λ	The number of sets	Actors
1	17	1: (3, 13); 2: (44, M009); 3: (R11, M016); 4: (M024, M025); 5: (M045, M046); 6: (M034, M051); 7: (6, M069); 8: (M019, M083); 9: (M078, M086); 10: (M079, M087); 11: (M080, M088); 12: (M028, M090); 13: (M084, M092); 14: (M098, M099); 15: (25, M101, M103, M104); 16: (47, 9, M017, M073, 14, 2, 37, 41, 59, 80, M003, M004, M011, M013, 15, 34, M010, 10, R10, 20, 62, R3, 26, M050, 4, 27, M057, M060, M064, M100, M107); 17: (8, M048, 75, R12, M026, M056, 30, M037, M065, M068, M108, M109, M110, M111)
2	4	1: (25, M101, M103, M104); 2: (14, 2, 37, 41, 59, 80, M003, M004, M011, M013, 15, 34, M010, 10, R10, 20, 62, R3, 26, M050, 4, 27, M057, M060, M064, M100, M107); 3: (75, R12, M026, M056); 4: (30, M037, M065)
3	3	1: (25, M101, M103, M104); 2: (41, 59, 80, M003, M004, M011, M013, 15, 34, M010, 10, R10, 20, 62, R3, 26, M050, 4, 27, M057, M060); 3: (M026, M056)
4	1	1: (15, 34, M010, 10, R10, 20, 62, R3, 26, M050, 4, 27, M057, M060)
5	1	1: (M010, 10, R10, 20, 62, R3, 26, M050, 4, 27, M057, M060)
6	1	1: (10, R10, 20, 62, R3, 26, M050, 4, 27, M057, M060)
8	1	1: (20, 62, R3, 26, M050, 4, 27, M057)
10	1	1: (26, M050, 4, 27, M057)
15	1	1: (4, 27, M057)
19	1	1: (27, M057)

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